An Overview of Properties of Fuzzy sets

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***Abstract*— This paper gives an overview and basic introduction to the fuzzy set theory and its properties . The definition of the difference set, algebraic product and algebraic sum of fuzzy set is shown. In addition, basic properties of those operations are described. Basic properties of fuzzy set are a little different from the properties of crisp set. The membership function is a crucial component of a fuzzy set. Therefore the operations with fuzzy sets are defined via their membership functions. We shall first present the concepts suggested by Zadeh in 1965[1] .Zadeh and other authors have suggested alternative or additional definitions for set-theoretic operations,**

1.INTRODUCTION

The first part of this paper covers the formal framework of the theory of fuzzy sets. It provides basic definitions of fuzzy sets and algebraic operations that will then serve for further considerations. It also extends the basic theory of fuzzy sets by introducing additional concepts and alternative operators .It introduces the extension principles, and covers fuzzy arithmetic. The number of disciplines in which fuzzy sets are applied is increasing steadily. business administration and management, chemistry, earth sciences, ecology and environmental science, economics, engineering (civil, industrial, mechanical, nuclear etc.), ergonomy, information technology, medi cine, social sciences, telecommunication, traffic management. Fuzzy applications in these areas are increasingly known by the terms "business intelligence" and "engineering intel ligence".

II.PRELIMINARIES

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.Fuzzy sets describe collections of objects whose boundaries are not precisely defined.

Definition :

A fuzzy set is a mapping f, from the universe of discourse, X, to the unit interval =[0, 1]. The value f(x) represents the degree to which x is a member of the fuzzy set given by f. This definition generalizes the identification of a crisp set with its characteristic function.attachments.

Let X be a space of points (objects), with a generic element of X denoted by x. Thus, X = {x}.

A fuzzy set (class) A in X is characterized by a mernbership (characteristic) function f (x) which associates with each point x in X a real number in the interval [0, 1], with the value of $f_A(x)$ at x representing the grade of membership of x in A. Thus, the nearer the value of f_A (x) to unity, the higher the grade of membership of x in A. For example if A is a set of element which are much greater than 1., Then, one can give a precise, characterization of A by specifying $f_A(x)$ as a function on R.Representative values of such a function might be: $f_A(x)= 0$; $f_A(1)= 0$; $f_A(5)=0.01$; $f_A(10)= 0.2$; $f_A(200)= 0.95$; $f_A(500) = 1$

A.Properties Of Fuzzy Sets

A fuzzy set is empty if and only if its membership function is identically zero on X.

Two fuzzy sets A and B are equal, written as A = B, if and only if

$f_A(x) = f_B(x)$ for all x in X .

Compliment: The complement of a fuzzy set A is denoted by A' and is defined by $f_{A'} = 1- f_A$ (1)

Containment: A is contained in B (or, equivalently, A is a subset of B,or A is smaller than or equal to B) if and only if $f_A <= f_B$. In symbols $A \subseteq B$ iff $f_A <= f_B$ (2)

Union: The union of two fuzzy sets A and B with respective membership functions $f_A(x)$ and $f_B(x)$ is a fuzzy set a, written as $C= A \cup B$, whose membership function is related to those of A and B by $f_c(x) = Max [f_A(x), f_B(x)]$ (3)

or, in abbreviated form

$f_c =f_A \vee f_B$ (4)

Note that $\cup$ has the associative property, that is,

$A \cup (B \cup C) =(A \cup B) \cup C.$

Comment. A more intuitively appealing way of defining the union is the following: The union of A and B is the smallest fuzzy set containing both A and B.

Lemma : D is any fuzzy set which contains both A and B, then it also contains the union of A and B.

Proof: To show that this definition is equivalent to (1), we note, first, that C is defined by (1) contains both A and B, since

$Max\ [f_A, f_B] >= f_A$

and

$Max\ [f_A , f_B] >= f_B$.

Furthermore, if D is any fuzzy set containing both A and B, then

$f_D .>= f_B$.

$f_D .>= f_A$.

and hence

$f_D >= Max[f_A,f_B] = fc$ which implies that C $\subset$ D.

Intersection: The intersection of two fuzzy sets A and B with respective membership functions fA(x) and fB(x) is a fuzzy set C, written as

C =A $\bigcap$ B, whose membership function is related to those of A and B by

$f_C(x)= Min[f_A(x),f_B(x)]$ (5)

or in abbreviated form

$fc=f_A \wedge f_B$ (6)

As in the case of the union, it is easy to show that the intersection of A and B is the largest fuzzy set which is contained in both A and B. As in the case of ordinary sets, A and .B are disjoint if A $\bigcap$ B is empty. Note that , like U, $\bigcap$ has the associative property.

B.Some Properties Of Union, Intersection And Complementation

With the operations of union, intersection, and complementation , it is easy to extend many of the basic identities which hold for ordinary sets to fuzzy sets. As examples, we have

$(A \bigcup B)' = A' \bigcap B'$ (7)

$(A \bigcap B)' = A' \bigcup B'$ (8)….

De M1organs laws

$C \bigcap (A \bigcup B) = (C \bigcap A) \bigcup (C \bigcap B)$ (9)

$C \bigcup (A \bigcap B) = (C \bigcup A) \bigcap (C \bigcup B)$

(10)………Distributive laws.

These and similar equalities can readily be established by showing that the corresponding relations for the membership functions of A, B, and C are identities. For example, in the case of (7), we have

$1 - Max[f_A,f_B]= Min\ [1-f_A,1-f_B]$ (11)

which can be easily verified to be an identity by testing it for the two possible cases: $f_A > f_B$ andj $f_A < f_B$

Similarly, in the case of (10), the corresponding relation in terms of f'' , f s , and fc is

$$Max\ [f_C, Min\ [f_A, f_B]] = Min\ [Max\ [f_C, f_A], Max\ [f_C, f_B]] \quad (12)$$

C. Interpretation For Unions And Intersections

In the case of ordinary sets, a set C which is expressed in terms of a family of sets A1, ….. , Ai, , ••• , An through the connectives $\bigcup$ and , can be represented as a network of switches $a_1,…, a_n$, with A_i , A_j and $A_i \bigcup A_j$ corresponding, respectively, to series and parallel combinations of a_i and a_j. More generally, a well-formed expression involving A_1, ••• , An $\bigcup$,and $\bigcap$ corresponds to a network of sieves $S_1(x)$, ••• , $S_n(x)$ which can be found by the conventional synthesis techniques for switching circuits. As a very simple example,

$C = [(A_1 \cup A_2) \cap A_3] \cup A_4$ (13)

conesponds to the network shown in Fig. 3.

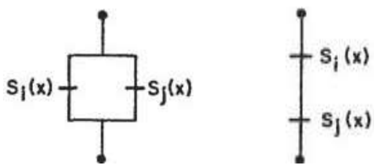

FIG 1. Parallel and series connection of sieves simultating $\bigcup$ and $\bigcap$

D.Algebraic Operations On Fuzzy Sets

In addition to the operations of union and intersection, one can define a number of other ways of forming combinations of fuzzy sets and relating them to one another. Among the more important of these are the following.

Algebraic product. The algebric product of A and B is denoted by AB and is defined in terms of the membership functions of A and B by the relation

Clearly, $f_{AB} = f_A f_B$. (14)

AB ⊂ A ⋂ B. (15)

Algebraic Sum.The algebrmic sum of A and B is denoted by A+B and is defined in terms of the membership functions of A and B by the relation

Provided the Sum $f_A + f_B$ is less than or equal to 1. Thus, unlike the algebraic product, the algebraic sum is meaningful only when the condition $f_A(x) + f_B(x) >= 1$ is satisfied for all x. (16)

Absolute difference. The absolute diference of A and B is denoted by $f_{|A - B|}$ and is defined by $f_{|A-B|} = |f_A - f_B|$

Note that in the case of ordinary sets IA - B I reduces to the relative complement of A ⋂ B in A ⋃ B.

EAlgebraic Product and Algebraic Sum

Let C be a non empty set and let h, g be membership functions of C . The functor h • g yielding a membership function of C is defined as follows:

Def1 : For every element c of C holds (h • g)(c) = h(c) • g(c). Let C be a non empty set and let h, g be membership functions of C . The functor h ⊕ g yielding a membership function of C is defined as follows:

Def. 2: For every element c of C holds (h ⊕ g)(c) = (h(c) + g(c)) − h(c) • g(c).

Let C be a non empty set, let h, g be membership functions of C , let A be a FuzzySet of C , h, and let B be a FuzzySet of C , g. The functor A • B yields a FuzzySet of C , h • g and is defined as follows:

Def.3 : A • B = [: C, (h • g)◦C :].
Let C be a non empty set, let h, g be membership functions of C , let A be a FuzzySet of C , h, and let B be a FuzzySet of C , g. The functor A ⊕ B yielding a FuzzySet of C , h ⊕ g is defined by:

Def 4 : A ⊕ B = [: C, (h ⊕ g)◦C :].

We now state a number of propositions:

(1) For every c holds (f • f)(c) <= f (c) and (f ⊕ f)(c)>= f (c).
(2) A • A ⊆ A and A ⊆ A ⊕ A.
(3) f • g = g • f and f ⊕ g = g ⊕ f.
(4) A • B = B • A and A ⊕ B = B ⊕ A.
(5) (f • g) • h = f • (g • h).
(6) (A • B) • D = A • (B • D).
(7) (f ⊕ g) ⊕ h = f ⊕ (g ⊕ h).
(8) (A ⊕ B) ⊕ D = A ⊕ (B ⊕ D).
(9) For every c holds (f • (f ⊕ g))(c) <= f (c) and (f ⊕ f • g)(c) >= f (c).
(10) A • (A ⊕ B) ⊆ A and A ⊆ A ⊕ A • B.
(11) (A ⊕ B) • (A ⊕ D) ⊆ A ⊕ B • D.
(12) (A • B)c = Ac ⊕ Bc.
(13) A ∪ B ⊆ A ⊕ B.
(14) A ⊕ (B ∪ D) = (A ⊕ B) ∪ (A ⊕ D) and A ⊕ B ∩ D = (A ⊕ B) ∩ (A ⊕ D).

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