Some properties of Fuzzy and anti Fuzzy Groups

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***Abstract*— In this paper some properties of an anti fuzzy subgroup of a group in relation to pseudo coset are investigated. It also establishes some results of a fuzzy subgroup μ of a group S using isomorphism theorems in relation to level subgroups of a fuzzy subgroup μ of a group S.**

1.INTRODUCTION

Major part of this work leans on the work of [5]. There are some new results using isomorphism theorems with some results in [5]. The concept of fuzzy sets was introduced by Zadeh in [12]. In [7], Rosenfeld .defined fuzzy subgroupoid and fuzzy subgroups in the following way.

Definition 1.1. Let S be a group. A fuzzy set μ of S is said to be a fuzzy subgroup of S if for all x, y in S

i) $\mu(x,v) \geq \min(\mu(x),\mu(y))$

ii) $\mu(x^{-1}) = \mu(x)$.

Further , fuzzy subgroups have been studied by Anthony and Sherwood in [6,7], by Sivaramakrishna Das in [11] and by Sherwood in [10]. We begin by giving some conventions, definitions, and propositions. Although some of these definitions may be found elsewhere, they are repeated here to help to the reader. Following this we define fuzzy normal subgroups, fuzzy level normal subgroups,anti fuzzy sub groups and their homomorphisms, and give some of their properties.

II.PRELIMINARIES

Definition 2.1 Let S be a non-empty set. A fuzzy subset μ of the set S is a function $\mu : S \to [0, 1]$.

Definition 2.2 Let S be a Group and μ a fuzzy subset of S. Then μ is called a fuzzy sub group of S if

(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$;

(ii) $\mu(x-1) = \mu(x)$;

(iii)μ is called a fuzzy normal sub group if $\mu(xy) = \mu(yx)$ for all x and y in S.

Definition 2.3: Let S be a group and μ a fuzzy subset of S. Then μ is called an anti fuzzy sub group of S if

(i) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$;

(ii) $\mu(x-1) = \mu(x)$.

Definition 2.4: Let μ and δ be any two fuzzy subsets of a set A . Then

(i) δ and μ are equal if $\mu(x) = \delta(x)$ for every x in A ;

(ii) δ and μ are disjoint if $\mu(x)= \delta(x)$ for every x in A.

(iii) $\delta \subseteq \mu$ if $\mu(x) \geq \delta(x)$.

Definition 2.5: Let μ be a fuzzy subset (sub group) of A . Then, for some d in [0, 1], the set $\mu_d = \{x \in A : \mu(x) \geq d\}$ is called a level subset (sub group) of the fuzzy subset (sub group) μ.

Remark 2.5.1: The set μ_d if it is group can be represented as $S^{\mu}{}_{d}$.

Definition 2.6: Let μ be a fuzzy subgroup of a group S. The set $H = \{x \in S : \mu(x) = \mu(e)\}$ is such that $o(\mu) = o(H)$.

Definition 2.7: Let μ be a fuzzy subgroup of a group S. μ is said to be normal if sup $\mu(x) = 1$ for all x in S. It is said to be normalized if there is an x in S such that $\mu(x) = 1$.

Definition 2.8 : Let S be a group and μ a fuzzy subset of S. Then μ is called an anti fuzzy subgroup of S if and only if $\mu(xy^{-1}) \leq \max\{\mu(x), \mu(y)\}$, and μ is called an anti fuzzy normal subgroup if $\mu(xy) = \mu(yx)$ for all x and y.

Definition 2.9: Let μ be a fuzzy subset of A . Then, for $t \in [1, 0]$, the set $\mu_t = \{x \in A : \mu(x) \leq t\}$ is called a lower level subset of the fuzzy subset μ.

Definition 2.10: Let μ be an anti fuzzy subgroup of A . Then, for $t \in [1, 0]$, the set $\mu_t = \{x \in A : \mu(x) \leq t\}$ is called a lower level subgroup of μ.

Definition 2.11: Let μ be an anti fuzzy subgroup of a group S of finite order. Then, the image of μ is $\text{Im}(\mu) = \{ti \in I : \mu(A) = ti$ for some x in S$\}$, where $I = [0, 1]$.

Definition 2.12: Let μ be an anti fuzzy subgroup of a group S. For a in S, the anti fuzzy coset aμ of S determined by a and μ is defined by $(a\mu)(x) = \mu(a^{-1}x)$ for all x in S.

Definition 2.13: Let μ be an anti fuzzy subgroup of a group S. For a and b in S, the anti fuzzy middle coset aμb of S is defined by (aμb)(x) = μ(a−1 xb−1) for all x in S.

Definition 2.14: Let μ be an anti fuzzy subgroup of S and an element a in S. Then pseudo anti fuzzy coset (aμ)p is defined by (aμ)p(x) = p(a)μ(x) for all x in S and p in P

Definition 2.15: The Cartesian product $\delta \times \mu : A \times Y \to [0, 1]$ of two anti fuzzy subgroups is defined by $(\delta \times \mu)(x, y) = \max\{\delta(x), \mu(y)\}$ for all (x, y) in A × Y and $R\delta$ is a binary anti fuzzy relation defined by $R\delta(x, y) = \max\{\delta\ \delta(x), \delta(y)\}$. The anti fuzzy relation $R\delta$ is said to be a similarity relation if

(i) $R\delta(x, x) = 1$;
(ii) $R\delta(x, y) = R(y,x)$;
(iii) $\max\{R\delta(x, y), R\delta(y, z)\} \le R\delta(x, z)$.

Definition 2.16: Let S be a finite group of order n and μ a fuzzy subgroup of S. Then for t1, t2 in [0, 1] such that t1 ≤ t2, μt2 ⊆ μt1 .. These are the images IEEE will scan and publish with your paper.

Definition 2.17: Let S be a finite group of order n and μ an anti fuzzy subgroup of S. Then for t1, t2 ∈ [0, 1] such that t1 ≤ t2, $\mu_{t1} \subseteq \mu_{t2}$.

Definition 2.18: Let f be a group homomorphism from a group S to H.Then there is an isomorphism φ : f (S) → S/Ker f , where φ is the canonical isomorphism associated with f .

Definition 2.19: Let S be a group and H , K normal subgroups of S such that H ≤ K . Then there is a natural isomorphism S/K ≅ (S/H)/(K/H).

Proposition 2.20: Let S be a group and μ a fuzzy subset of S. Then μ is a fuzzy subgroup of S if and only if St is a level subgroup of S for every t in [0, μ(e)], where e is the identity of S.

Proposition 2.21: H as described in 2.6 can be realized as a level subgroup.

Theorem 2.22: S is a Dedekind or Hamiltonian group if and only if every fuzzy subgroup of S is fuzzy normal subgroup. (A Dedekind and Hamiltonian groups have all the subgroups to be normal).

III. Briefly on Properties of Anti Fuzzy Subgroup

Proposition 3.1: Any two pseudo cosets of an anti fuzzy subgroup of a group S are either identical or disjoint.

Proof : Assume that (aμ)p and (bμ)p are any two identical pseudo anti fuzzy cosets of μ for any a and b in S. Then, (aμ)p(x) = (bμ)p(x) for all x in S. Assume also on the contrary that they are disjoint. Then, there is no y in S such that (aμ)p(y) = (bμ)p(y) which implies that p(a)μ(y) = p(b)μ(y). The consequence is that p(a) = p(b). This makes the assumption (aμ)p (x) = (bμ)p(x) false.

Conversely, assume that (aμ)p and (bμ)p are disjoint, then p(a)μ(y) = p(b)μ(y) for every y in S. But if it is assumed that this is also identical, then p(a)μ(y) = p(b)μ(y) and that means p(a) = p(b) so that p(a)μ(y) = p(b)μ(y) cannot be true.

Proposition 3.2: Let μ be an anti fuzzy subgroup of any group S. Let $\{\mu_i\}$ be a partition of μ. Then
(i) each μ_i is normal if μ is normalized;

(ii) each μ_i is normal if μ is normal.

Proof: Note that for each i, $\mu_i \subseteq \mu$ which implies that $\mu_i(x) \le \mu(x)$ for all x in S.

(i) Since μ is normalized, there is an x_0 in S such that $\mu_i(x) \le \mu(x) \le \mu(x_0) = 1$ for each i. Whence, $\mu_i(x) \le 1$. Then $\sup \mu_i(x) = 1$.

(ii) Since μ is normal, sup μ(x) = 1, then μ(x) ≤ 1. Note that $\mu_i(x) \le \mu(x) \le 1$.Then $\mu_i(x) \le 1$ and $\sup \mu_i(x) = 1$.

Proposition 3.3: Let μ be an anti fuzzy subgroup of any group S. Then μ(e) ≤ 1 even if μ is normalized.

Proof: Note that for all x in S, 0 ≤ μ(x) ≤ 1.
$\mu(e) = \mu(xx^{-1}) \le \max\{\mu(x), \mu(x^{-1})\} = \mu(x)$ since
$\mu(x) = \mu(x^{-1})$ for all x in S.

But since μ is normal, there is an x_0 in S such that
$\mu(e) \le \mu(x) \le \mu(x_0) = 1$. Hence $\mu(e) \le 1$.

Proposition 3.4: Let μ be an anti fuzzy subgroup of any group S and Rμ : S × S → [0, 1] be given by $R\mu(x, y) = \mu(xy^{-1})$. Rμ is not a similarity relation.

Proof The reference [4] has shown that this is a similarity relation when μ is a fuzzy subgroup of S. But

$R_\mu (x, x) = \mu(xx^{-1}) = \mu(e) \leq 1$. R_μ is not symmetric, hence not a similarity relation

IV. Application of Isomorphism Theorems of Groups to Fuzzy Subgroups

Proposition 4.1 Let f be a group homomorphism between S and H . Let μ be a fuzzy subgroup of H . Then S is isomorphic to a level subgroup of H .

Proof Since f is a homomorphism, it is defined on S.
$\text{Ker } f = \{x \in S : f(x) = e_H\} \Leftrightarrow \{x \in S : \mu f(x) = \mu(e_H) \leq 1\}$.

Hence, $\mu f(A) \leq 1$ for all A in S since μ is a fuzzy subgroup of H and f (x) is in H .

Ker f = S so that $\mu f(S) \leq 1$.
Also, note that
$f(S) = \{y = f(x) \in H : \mu f(x) = \mu(y) = \mu(e_H)\}$.

By 2.21 and 2.6, f (S) is a level subgroup, say H_t^{μ} of H .

$S/S = S \cong H^{\mu}{}_t$

by Definition 2.18.

Remark 4.2: It can be said then that every group S is isomorphic to a level subgroup of a group H if there is a group homomorphism between S and H and μ a fuzzy subgroup of H exist.

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