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# SINGULARITY PROBLEM & ITS SOLUTION FOR 6 DEGREE OF FREEDOM INDUSTRIAL ROBOTS

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***ABSTRACT*— Singularity problem is the most common problem occurring in 6 degree of freedom industrial robots. It is related to singular matrix concept of mathematics. A matrix is said to be singular if the determinant of that matrix is 0. There are various cases where a matrix becomes singular. The research paper focuses on the singularity problem & its solution for 6 degree of freedom industrial robots.**

## I. INTRODUCTION:

A body moving freely in space has 6 degrees of freedom i.e. 3 degree of freedom for the position in space defined by the coordinates X, Y & Z & 3 degree of freedom for the orientation in space defined by the angles of rotation A, B & C. This means that the robot can reach any point in the workspace with any orientation.

Before understanding the singularity problem in industrial robots, first we need to understand what is a singular matrix & other mathematical terms related to matrix.

### A. WHAT IS A MATRIX?

A matrix is a array of number arranged in dimension m x n, where;

m = number of rows
n = number of columns

A square matrix is a matrix where m = n i.e. Number of rows = Number of columns. The square matrix could be of size [1 x 1], [2 x 2], [3 x 3], etc…

A singular matrix is a matrix whose determinant is 0[4][5][6]. A singular matrix is also defined as a matrix which cannot have an inverse matrix. To understand it consider a 2 x 2 matrix with the following cases:

$$\begin{bmatrix}0&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}0&0\\1&0\end{bmatrix}, \begin{bmatrix}0&0\\1&1\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}$$

$$\begin{bmatrix}0&1\\0&1\end{bmatrix}, \begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}1&0\\1&0\end{bmatrix}, \begin{bmatrix}1&1\\0&0\end{bmatrix}, \begin{bmatrix}1&1\\1&1\end{bmatrix}.$$

In all of the above cases the determinant value of the matrix is 0, also known to as singular matrix. Similar problem occurs when we consider a 3 x 3 matrix if it's a singular matrix[1][2][3].

Consider a 3 x 3 matrix:

$$\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{2}&\mathbf{2}&\mathbf{2}\\\mathbf{3}&\mathbf{3}&\mathbf{3}\end{bmatrix}$$

If we have the determinant of this matrix than:

$$\begin{vmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{2}&\mathbf{2}&\mathbf{2}\\\mathbf{3}&\mathbf{3}&\mathbf{3}\end{vmatrix}=\mathbf{0}$$

Thus the matrix is a singular matrix.

## II. SINGULARITY PROBLEM & ITS SOLUTION FOR 6 DOF INDUSTRIAL ROBOT:

A 6 degree of freedom robot means it has 6 axis specific motors termed as a1, a2, a3, a4, a5, a6. In the case of robot kinematic systems there are certain points in space that give rise to so-called singularities. A singularity is characterized by the fact that two axes of the robot are collinear (aligned in a straight line). A typical configuration with a singularity is the overhead position of the tool. In this case, axis 1 and axis 6 are aligned. The controller cannot unambiguously assign a rotation about the vertical to axis 1 or axis 6. Another situation is when axis 5 passes through the zero position. In this case,axis 4 and axis 6 are aligned. Here, there is an infinite number of different axis positions that result in the same tool position or an infinite number of different motion paths in which multiple axes would have to be moved against each other at infinite velocity. Some controllers simply abort the program when passing through such a point[7].

A singularity position is characterized by the fact that unambiguous reverse transformation i.e conversion of Cartesian coordinate system to axes specific value is not possible, even if they are specified. In this case, or if very slight Cartesian coordinate causes large variations to the axis angles, than singularity positions occurs. This is a mathematical property, not a mechanical one, and thus only exists for CP motions and not axis motions[7]. If we consider a matrix as follows:

| | A4 (A) | A5 (B) | A6 (C) |
|---|---|---|---|
| A1 (X) | 1 | 0 | 1 |
| A2 (Y) | 1 | 1 | 1 |
| A3 (Z) | 1 | 1 | 1 |

In the above case the point of intersection a1 & a5 axes is 0. This means that there is an singularity problem at that point. Such type of singularity is also known to as Overhead Singularity (α1 position) alos referred to as Sitting On Head Problem wherein axes 5 is perpendicular to axes a1 i.e. axes a5 is overhead to axes a1. This problem is referred to as α1 position. In the overhead singularity, the wrist root point i.e. the center point of axes A5 is located vertically above axes A1 of the robot[7]. The position of axes A1 can be undetermined by means of reverse transformation and can thus take any value.

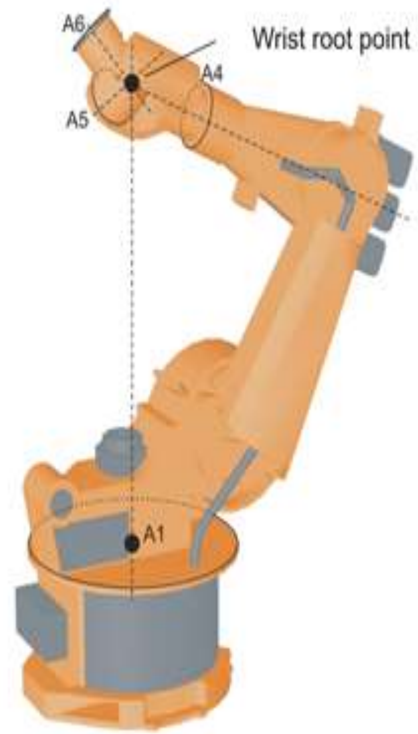


**FIG.1: OVERHEAD SINGULARITY (α1 POSITION)**

The solution to this problem is to move the axes a5 such that the center point of a1 & a5 mismatches each other & can overcome singularity problem of α1 position[7].

Now let's consider a matrix as follows:

| | A4 (A) | A5 (B) | A6 (C) |
|---|---|---|---|
| A1 (X) | 1 | 1 | 1 |
| A2 (Y) | 0 | 0 | 1 |
| A3 (Z) | 0 | 0 | 1 |

In the above case the point of intersection of axes a2, a3 & a5 are 0 i.e. the center point of the position of axes a2, a3 & a5 are aligned in a straight path due to which the point of intersection of this axes is 0 resulting to singularity problem termed as Extended Position Singularity (α2 position), also referred to as Superman Position. In the extended position, the wrist root point i.e. center point of axes A5 is located in the extension of axes A2 and A3 of the robot[7]. The robot has reached the limit of its work envelope. Even though reverse transformation does provide unambiguous axis angles, low Cartesian velocity that results into high axes velocity for axes A2 and A3.

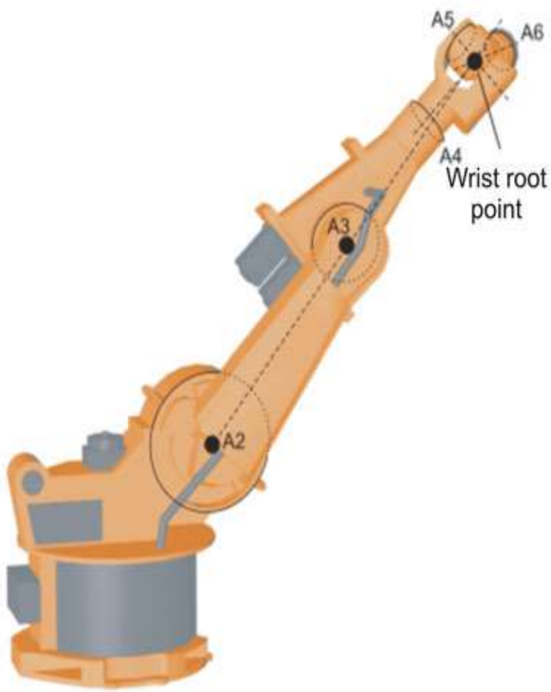


**FIG.2: EXTENDED SINGULARITY POSITION (α2 POSITION)**

The solution to this problem is to either move axes a3 or a5 such that their center point reference mismatches each other & overcome the singularity problem. In the above case either axes a3 shall be moved or either axes a5 shall be moved to overcome the α2 position.

Now let's consider a matrix as follows:

| | A4 (A) | A5 (B) | A6 (C) |
|---|---|---|---|
| A1 (X) | 0 | 0 | 0 |
| A2 (Y) | 1 | 1 | 1 |
| A3 (Z) | 1 | 1 | 1 |

In the above case the reference points of axes a4, a5 & a6 becomes 0 i.e. the wrist axes of the arm are aligned to each other such that axes 5 center matches with the center of axes a4 & a6 resulting in Wrist Axes Singularity (α5 Position) also referred to as Snake Trap Problem wherein axes a5 is trapped between the axes a4 & a6. In the wrist axis singularity position, A4 and A6 axes are parallel to each with regards to axes A5, it is in the range of ± 0.01812°[7]. The position of the two axes can be undetermined unambiguously by reverse transformation. There can be infinite number of possible axes positions for axes A4 and A6 to become identical axes angle & sums at this event of occurrence.

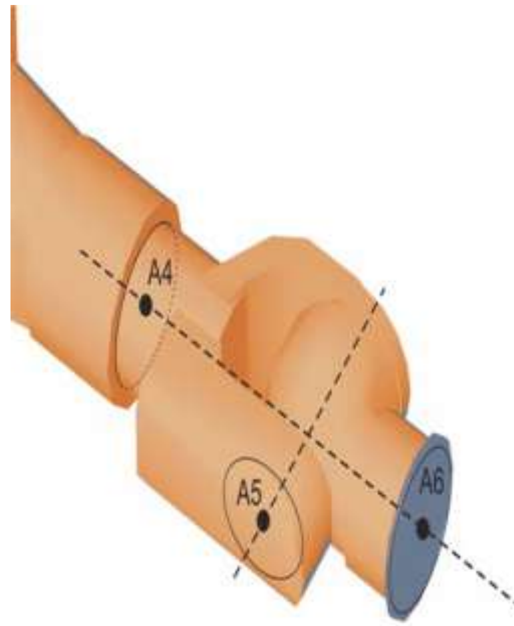


**FIG.3: WRIST AXES SINGULARITY (α5 POSITION)**

The solution to this problem is either move a5 axes or a6 axes or a4 axes such that their center point of reference mismatches with each other. Any of the 3 axes movement or slight variation in all of the 3 movement of the axes can overcome the α5 singularity problem.

## III. CONCLUSION:

The above discussion clears that singularity problem is a common problem which occurs in 6 degree of freedom robot & can be easily overcome by slight variation in the axes movement to mismatch the reference point of respective axes. Also it is to be noted that singularity problem itself is safety stop which avoids the robot to operate in any of the unfavorable conditions happening knowingly or unknolwingly as in all of the singularity problem the differential load is directly applied to either axes a1, a2 or a5 respectively, which can damage the particular axes if operated with such singularity problem & at the same time by having slight variation in the movement of respective axes to mismatch the reference point & distribute the differential load equally, singularity problem can be overcome & 6 degree of freedom robot can operate smoothly.

## IV. REFERENCES:

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